Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

Quiz 9 – 11/22/2023

Problem	Weight	Score
1	1	
2	0.5	
3	0.5	
Total		/ 20

Instructions. You have the entire class to complete this quiz. You may use your own course materials, as well as any materials directly linked from the course website. You may use your plebe-issue calculator. You may collaborate with other students on this quiz.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem 1. Customers call the reservation desk at Fluttering Duck Airlines at a rate of 4 customers per hour. There is 1 agent working at the reservation desk at any given time, and each phone call takes an average of 20 minutes. The phone system can only handle 3 customers at a time (1 with an agent, 2 waiting) – any phone calls arriving when there are 3 customers are simply lost. This setting can be modeled as a birth-death process with the following arrival and service rates:

$$\lambda_i = \begin{cases} 4 & \text{if } i = 0, 1, 2 \\ 0 & \text{if } i = 3, 4, \dots \end{cases} \qquad \mu_i = 3 \quad \text{for } i = 1, 2, \dots$$

a. Over the long run, what is the probability that there are n customers in the system? ($n = 0, 1, 2, 3$)

See Example 1 in Lesson 15, as well as Problems 2b and 3a in the Lesson 15 Exercises for similar examples.

b. Suppose you find that the steady-state probabilities are:

$$\pi_0 = 0.15 \qquad \pi_1 = 0.21 \qquad \pi_2 = 0.27 \qquad \pi_3 = 0.37 \qquad \pi_n = 0 \quad \text{for } n = 4, 5, \dots$$

(This may or may not match what you found in part a.) Over the long run, what is the expected number of customers in the system?

Note that the problem asks for the expected number of customers in the system ℓ, not the expected number of customers in the queue ℓ_q. See the top of page 4 of Lesson 15 for the definitions of ℓ and ℓ_q.

See Example 2 in Lesson 15, as well as Problems 2c and 3b in the Lesson 15 Exercises for examples of computing ℓ_q, which is similar to computing ℓ.

c. Suppose you find that the long-run expected number of customers in the system is 1.86. (This may or may not match what you found in part b.) Over the long run, what is the expected waiting time? Use the steady state probabilities given above in part b.

Some of you did this problem by first computing ℓ_q, then computing w_q using the queue-only version of Little's law, and then finally using the fact that $w = w_q + 1/\mu$ when the service rate is a constant μ. This works, but is more complicated than necessary.

You were given that $\ell = 1.86$ in the problem. You can then find w through the system-wide version of Little's law. See page 5 of Lesson 15 for details.